CBCS SCHEME

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17MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that,
$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$$ **(06 Marks)**

b. If $\vec{A}=i+2j+3k$, $\vec{B}=-i+2j+k$, $\vec{C}=3i+j$, find the value of P such that $\vec{A}+P\vec{B}$ is perpendicular to $\vec{C}$. **(07 Marks)**

c. Show that the points whose position vectors are $3i-2j+4k$, $6i+3i+k$, $5i+7j+3k$ and $2i+2j+6k$ are coplanar. **(07 Marks)**

OR

2 a. Express : $\frac{(1+i)(2+i)}{(3+i)}$ in the form $x+iy$. **(06 Marks)**

b. Simplify : $\frac{(\cos 6\theta-i\sin 6\theta)^3(\cos 2\theta+i\sin 2\theta)^7}{(\cos 4\theta-i\sin 4\theta)^3}$ **(07 Marks)**

c. Show that : $\left[\vec{a}\times\vec{b}\,,\,\vec{b}\times\vec{c},\,\vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$ **(07 Marks)**

Module-2

3 a. If $y=e^{m\cos^{-1}x}$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$ **(06 Marks)**

b. Prove that the pair of curves $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$ intersect orthogonally. **(07 Marks)**

c. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u=x^2+y^2+z^2$, $v=xy+yz+zx$, $w=x+y+z$. **(07 Marks)**

OR

4 a. Prove with usual notations:
$$\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$ **(06 Marks)**

b. Obtain the Maclaurin's series expansion of $\log(1+x)$ upto third degree terms. **(07 Marks)**

c. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $xu_x+yu_y=\sin 2u$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi/6}\cos^4(3x)\sin^2(6x)dx$ **(06 Marks)**

b. Evaluate $\int_0^1\int_x^{\sqrt{x}}xy\,dy\,dx$ **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$ **(07 Marks)**

OR

6 a. Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4}\,dx$ **(06 Marks)**

b. Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dy\,dx$ **(07 Marks)**

c. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dx\,dy\,dz$ **(07 Marks)**

Module-4

7 a. A particle moves along a curve whose parametric equations are :
$x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$. **(06 Marks)**

b. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along $2i - 3j + 6k$. **(07 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

8 a. Find the angle between the tangents to the curve :
$\vec{r} = \left(t - \frac{t^3}{3}\right)i + t^2 j + \left(t + \frac{t^3}{3}\right)k$ at $t = \pm 3$. **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$ find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ at the point $(1, -1, 1)$. **(07 Marks)**

c. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla\cdot(r^n\vec{r}) = (n+3)\,r^n$. **(07 Marks)**

Module-5

9 a. Solve : $y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$ **(06 Marks)**

b. Solve : $(x^2 + y)\,dx + (y^3 + x)\,dy = 0$ **(07 Marks)**

c. Solve : $\frac{dy}{dx} + \frac{y}{x} = y^2x$ **(07 Marks)**

OR

10 a. Solve : $\frac{dy}{dx} = \frac{3x+2y-5}{2x-3y+5}$ **(06 Marks)**

b. Solve : $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$ **(07 Marks)**

c. Solve : $y(2xy+1)dx - xdy = 0$ **(07 Marks)**

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